Aproximación de π

Sea $k \in \mathbb{N} \cup \{0\}$. Para un polígono regular de n lados inscrito en una circunferencia se tiene:

n	perímetro/diámetro
3	$3\sqrt{3}/2$
6	3
12	$6\sqrt{2-\sqrt{3}}$
24	$12\sqrt{2-\sqrt{2+\sqrt{3}}}$
$\vdots$	$\vdots$
$3 \cdot 2^k$	$3 \cdot 2^{k-1}\sqrt{2-a_k}$
$\vdots$	$\vdots$
∞	π

n	perímetro/diámetro
4	$2\sqrt{2}$
8	$4\sqrt{2-\sqrt{2}}$
16	$8\sqrt{2-\sqrt{2+\sqrt{2}}}$
32	$16\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}$
$\vdots$	$\vdots$
$4 \cdot 2^k$	$4 \cdot 2^{k-1}\sqrt{2-b_k}$
$\vdots$	$\vdots$
∞	π

n	perímetro/diámetro
5	$5\sqrt{(5-\sqrt{5})/2}/2$
10	$5\sqrt{2-\sqrt{(3+\sqrt{5})/2}}$
20	$10\sqrt{2-\sqrt{2+\sqrt{(3+\sqrt{5})/2}}}$
40	$20\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{(3+\sqrt{5})/2}}}}$
$\vdots$	$\vdots$
$5 \cdot 2^k$	$5 \cdot 2^{k-1}\sqrt{2-c_k}$
$\vdots$	$\vdots$
∞	π

Donde,

$$a_k = \begin{cases} -1, & k = 0 \\ \sqrt{2 + a_{k-1}}, & k > 0 \end{cases} \qquad b_k = \begin{cases} 0, & k = 0 \\ \sqrt{2 + b_{k-1}}, & k > 0 \end{cases} \qquad c_k = \begin{cases} \frac{\sqrt{5}-1}{2}, & k = 0 \\ \sqrt{2 + c_{k-1}}, & k > 0 \end{cases}$$

Además,

$$\sin\frac{\pi}{3\cdot 2^k} = \frac{\sqrt{2-a_k}}{2} \qquad \sin\frac{\pi}{4\cdot 2^k} = \frac{\sqrt{2-b_k}}{2} \qquad \sin\frac{\pi}{5\cdot 2^k} = \frac{\sqrt{2-c_k}}{2}$$

$$\cos\frac{\pi}{3\cdot 2^k} = \frac{\sqrt{2+a_k}}{2} \qquad \cos\frac{\pi}{4\cdot 2^k} = \frac{\sqrt{2+b_k}}{2} \qquad \cos\frac{\pi}{5\cdot 2^k} = \frac{\sqrt{2+c_k}}{2}$$